

# Long rainbow cycles in proper edge-colorings of complete graphs \*

András Gyárfás, Miklós Ruszinkó<sup>†</sup>

Computer and Automation Research Institute  
Hungarian Academy of Sciences  
Budapest, P.O. Box 63  
Budapest, Hungary, H-1518  
[gyarfas,ruszinko@sztaki.hu](mailto:gyarfas,ruszinko@sztaki.hu)

Gábor N. Sárközy<sup>‡</sup>

Computer Science Department  
Worcester Polytechnic Institute  
Worcester, MA, USA 01609  
[gsarkozy@cs.wpi.edu](mailto:gsarkozy@cs.wpi.edu)

and

Computer and Automation Research Institute  
Hungarian Academy of Sciences  
Budapest, P.O. Box 63  
Budapest, Hungary, H-1518

Richard H. Schelp<sup>§</sup>

University of Memphis  
Department of Mathematical Sciences  
Memphis, TN 38152, USA  
[rschelp@memphis.edu](mailto:rschelp@memphis.edu)

*Dedicated to the memory of our colleague and friend Dick Schelp.*

---

\*2000 Mathematics Subject Classification: 05C70, 05C38, 05C15.

<sup>†</sup>Research supported in part by OTKA Grant No. K68322.

<sup>‡</sup>Research supported in part by OTKA Grant No. K68322, by NSF Grant DMS-0968699 and by a Janos Bolyai Research Scholarship.

<sup>§</sup>Research supported by the FIST (Finite Structures) project, in the framework of the European Community's "Structuring the European Research Area programme".

## Abstract

We show that any properly edge-colored  $K_n$  contains a rainbow cycle with at least  $(4/7 - o(1))n$  edges. This improves the lower bound of  $n/2 - 1$  proved in [1].

We consider *properly edge-colored* complete graphs  $K_n$ , where two edges with the same color cannot be incident to each other, so each color class is a matching. An important and well investigated special case of proper edge-colorings is a *factorization* where each color class forms a perfect (if  $n$  is even) or nearly perfect (if  $n$  is odd) matching. A colored subgraph of  $K_n$  is called *rainbow* if its edges have different colors.

The size of rainbow subgraphs of maximum degree two, i.e. union of paths and cycles in proper colorings are well investigated. A consequence of Ryser's well-known conjecture ([12] stating that every Latin square has a transversal) would be that for odd  $n$  in every factorization of  $K_n$  there is a rainbow 2-factor (and for even  $n$  a 2-factor covering all but one vertices). Although this is not known, there were several results that made advances towards Ryser's conjecture and show the existence of a 2-factor covering  $n - o(n)$  vertices, [4, 10, 13, 14]. Andersen [3] applied the method of [4] to prove that in every proper coloring of  $K_n$  there is a rainbow subgraph with at least  $n - \sqrt{2n}$  vertices whose components are paths.

Another line of research looked for rainbow Hamiltonian cycles from the assumption that there is an upper bound  $k$  on the number of colors in each color class. This problem is mentioned in Erdős, Nešetřil and Rödl [5]. Hahn and Thomassen [9] showed that  $k$  could grow as fast as  $n^{1/3}$  and in fact Hahn conjectured (see [9]) that the growth of  $k$  could be linear in  $n$ . After further improvements [7], Albert, Frieze and Reed [2] proved the Hahn Conjecture by showing that  $k$  could be  $\lceil cn \rceil$ , for any constant  $c < 1/32$  if  $n \geq n_0(c)$ . See also [6] for related results.

Although it is widely believed that in every proper coloring of  $K_n$  there is a rainbow path and cycle with length almost  $n$  (the obstacle to a spanning rainbow path or cycle comes from a special factorization, see [1], [9], [11]), the above mentioned results do not imply such a bound. As far as we know the best lower bounds are  $2n/3$  for the path ([8]) and  $n/2 - 1$  for the cycle ([1]). The purpose of this note is the improvement of the latter result to  $(1 - o(1))\frac{4n}{7}$ .

**Theorem 1.** *For arbitrary  $\varepsilon$ , where  $1/2 > \varepsilon > 0$ , there exists an  $n_0(\varepsilon)$  such that if  $n \geq n_0(\varepsilon)$ , then in any proper edge-coloring of  $K_n$  there is a rainbow cycle with length at least  $(\frac{4}{7} - \varepsilon)n$ .*

**Proof:** The vertex-set and the edge-set of a graph  $G$  are denoted by  $V(G)$  and  $E(G)$ .  $C_l$  is the cycle with  $l$  vertices and  $P_l$  is the path with  $l$  vertices.

Fix  $\varepsilon$ , such that  $1/2 > \varepsilon > 0$  and choose constants  $d = d(\varepsilon)$  and  $n_0 = n_0(\varepsilon)$  in the following way:

$$d = d(\varepsilon) = \left(\frac{48}{7\varepsilon}\right)^2, \quad n_0 = n_0(\varepsilon) = \frac{8(d+1)}{\varepsilon}. \quad (1)$$

Assume that  $n \geq n_0$ . Let us take an arbitrary proper edge-coloring of  $K_n$  and let  $C_t = \{v_1, \dots, v_t\}$  be a rainbow cycle with  $t$  edges such that  $t$  is maximum. We will show that

$$t \geq \left(\frac{4}{7} - \varepsilon\right)n.$$

During the proof we will try to increase the length of  $C_t$  using rainbow “detours”. More precisely a segment of the cycle  $C_t$  will be deleted and replaced by a new part. If the vertices added to the cycle are greater in number than those removed, a longer rainbow cycle is obtained contradicting the fact that  $C_t$  has maximum length. The colors already used on  $C_t$  will be called old colors and the set containing them will be denoted by *OLD*. The colors not used yet are called new colors and the set containing them will be denoted by *NEW*, i.e., we start with  $OLD = \{\text{colors used along } C_t\}$  and *NEW* consists of the remaining colors. These sets of colors, however, may vary during the proof according to the detours along which we will try to enlarge  $C_t$ . For  $x \in V$ ,  $R \subseteq V$  we denote by  $\deg_{NEW}(x, R)$  the number of edges adjacent to  $x$  and  $u \in R$  having color from *NEW*.

To make the presentation more transparent, we avoid using floors and ceilings. Since the obtained result is probably far from the best possible these “inaccuracies” do not have any impact.

**Case 1:** There exists a pair of vertices  $y_1$  and  $y_2$  in  $C_t$  which are within distance  $d$  along the cycle and which are adjacent to two different vertices, say  $x_1$  and  $x_2$ , in two different new colors in the remaining part of the vertex set  $R = V \setminus V(C_t)$ . Here we will try to delete this short segment of  $C_t$  between  $y_1$  and  $y_2$  and replace it with a longer rainbow path, as outlined above. Move the two new colors used on the edges  $(x_1, y_1)$  and  $(x_2, y_2)$  from *NEW* to *OLD*. Notice, that no vertex  $x \in R$  is connected to two consecutive  $v_i, v_{i+1}$  vertices along  $C_t$  in new colors, since otherwise we obtain a longer cycle by substituting the edge  $(v_i, v_{i+1})$  by the path  $P = \{v_i, x, v_{i+1}\}$ . Therefore, for an arbitrary vertex  $x \in R$

$$\deg_{NEW}(x, R) \geq n - t - 1 - t/2 - 2 = n - 3t/2 - 3. \quad (2)$$

Next we find a rainbow path  $P_d$  with  $d$  vertices in  $R$  in new colors starting at  $x_1$  and avoiding  $x_2$ . This is always possible assuming

$$\deg_{NEW}(x, R) - 2d \geq n - 3t/2 - 3 - 2d \geq 0, \quad \text{i.e.,} \quad t \leq 2n/3 - 4d/3 - 2.$$

Let  $x'_1$  be the other endpoint of  $P_d$  and set  $R' = R \setminus (V(P_d) \setminus x'_1)$ . Move the colors along the path  $P_d$  from *NEW* to *OLD*, i.e.,

$$|NEW| \geq n - t - 2 - d.$$

Similar to (2), for an arbitrary vertex  $x \in R'$

$$\deg_{NEW}(x, R') \geq n - 3t/2 - 3 - (2d - 1) = n - 3t/2 - 2d - 2, \quad (3)$$

where we have to subtract  $d-1$  colors used in  $P_d$  and  $d$  other colors (possibly) going to vertices in  $P_d$ . Let  $N_{NEW}(x, R')$  be the set of those vertices in  $R'$  which are adjacent to  $x$  in new colors. Set

$$\Gamma_1 = N_{NEW}(x'_1, R'), \Gamma_2 = N_{NEW}(x_2, R').$$

If there exists an  $z \in \Gamma_1 \cap \Gamma_2$ , then we could substitute the path  $\{y_1, \dots, y_2\}$  of length  $\leq d$  along the cycle by the path  $\{y_1, x_1, P_d, x'_1, z, x_2, y_2\}$  of length  $> d$  and obtain a longer rainbow cycle, a contradiction. So assume  $\Gamma_1 \cap \Gamma_2 = \emptyset$ . Then, since  $|R'| = n - t - d + 1$ , for

$$S = R' \setminus (\Gamma_1 \cup \Gamma_2),$$

by (3) we have

$$|S| \leq n - t - d + 1 - 2(n - 3t/2 - 2 - 2d) = 2t - n + 3d + 5.$$

Without loss of generality we may assume that  $|\Gamma_1| \geq |\Gamma_2|$  and then, clearly,

$$\frac{n - t - d + 1}{2} \leq \frac{|R'|}{2} \leq |\Gamma_1 \cup S|,$$

and by (3)

$$|\Gamma_1 \cup S| \leq |R'| - (n - 3t/2 - 2 - 2d) = n - t - d + 1 - (n - 3t/2 - 2 - 2d) = \frac{t}{2} + d + 3.$$

Notice, that if  $x \in \Gamma_1$  is adjacent to a vertex  $z \in \Gamma_2$  in a new color then  $(x'_1, x)$  and  $(x_2, z)$  must have the same color. Otherwise we could substitute the path of length  $\leq d$   $\{y_1, \dots, y_2\}$  in the cycle by the path  $\{y_1, x_1, P_d, x'_1, x, z, x_2, y_2\}$  of length  $> d$  and obtain a longer rainbow cycle, a contradiction. And since the coloring is proper, every  $x \in \Gamma_1$  is adjacent to at most one vertex  $z \in \Gamma_2$  in a new color. Therefore, every vertex  $x \in \Gamma_1$  has all but at most one of its neighbors in new colors in  $\Gamma_1 \cup S$ , i.e., by (3) for every  $x \in \Gamma_1$

$$\deg_{NEW}(x, \Gamma_1 \cup S) \geq n - 3t/2 - 2d - 3.$$

If twice this degree is greater than  $|\Gamma_1 \cup S| + 3$ , i.e.,

$$2(n - 3t/2 - 2d - 3) \geq \frac{t}{2} + d + 6 \geq |\Gamma_1 \cup S| + 3, \quad (4)$$

then two arbitrary vertices in  $\Gamma_1$  can be joined by 3 different paths of length two in new colors. If (4) does not hold, then we have

$$t > \frac{4n}{7} - \frac{10d + 24}{7}, \quad (5)$$

i.e., the original cycle is sufficiently large. Therefore, we will assume that two arbitrary vertices in  $\Gamma_1$  can be joined by three paths of length two in new colors.

To finish this case we will try to find two vertices of distance 1, 2 or 3, say  $v_i$  and  $v_j$ ,  $|j - i| \leq 3$ , along the cycle such that they are adjacent to two different vertices  $x_i, x_j \in \Gamma_1$  in two different new colors. If such two edges exist, then one of the 3 existing paths, say  $P$ , of length 2 between  $x_i$  and  $x_j$  in new colors contains neither the color of the edge  $(v_i, x_i)$ , nor the color of the edge  $(v_j, x_j)$ . Replacing the path of length  $\leq 3$   $\{v_i, \dots, v_j\}$  by the path  $\{v_i, x_i, P, x_j, v_j\}$  of length four we obtain a longer rainbow cycle, a contradiction.

Notice that every  $x \in \Gamma_1$  satisfies

$$\begin{aligned} \deg_{NEW}(x, C_t) &\geq |NEW| - |P_d| - (|\Gamma_1 \cup S| - 1) - 1 \geq n - t - 2d - 4 - (|\Gamma_1 \cup S| - 1) \geq \\ &\geq n - t - 2d - \left(\frac{t}{2} + d + 3\right) - 3 = n - \frac{3t}{2} - 3d - 6, \end{aligned} \quad (6)$$

and therefore, for the number of edges in new colors  $|E_{NEW}[\Gamma_1, C_t]|$  in the bipartite graph with parts  $C_t$  and  $\Gamma_1$  by (3) and (6) we have

$$|E_{NEW}[\Gamma_1, C_t]| \geq \deg_{NEW}(x'_1, R') \cdot (n - \frac{3t}{2} - 3d - 6). \quad (7)$$

Next we get an upper bound for the number of these edges with respect to the degrees of the vertices in  $C_t$ . In order to have this, partition the vertices along  $C_t$  into consecutive quadruples  $\{v_1, v_2, v_3, v_4\}, \{v_5, v_6, v_7, v_8\}, \dots$ . (If 4 does not divide  $t$  then let the last part contain one, two or three vertices.)

**Claim 2.** *If for some  $i$  and for some quadruple  $\{v_{4i+1}, v_{4i+2}, v_{4i+3}, v_{4i+4}\}$  the sum of the degrees*

$$s_i = \sum_{j=1}^4 \deg_{NEW}(v_{4i+j}, \Gamma_1) \geq |\Gamma_1| + 3, \quad (8)$$

*then there exist  $v_{4i+k}$  and  $v_{4i+l}$ ,  $1 \leq k < l \leq 4$ , such that they are adjacent to two different vertices  $x_i$  and  $x_j$  in  $\Gamma_1$  in two different new colors.*

**Proof.** Indeed, if (8) holds then there have to be vertices in  $\Gamma_1$  which are covered 2 or 3 times by the four sets in

$$T_i = \bigcup_{j=1}^4 N_{NEW}(v_{4i+j}, \Gamma_1).$$

If  $\exists x \in \Gamma_1$  which is covered 3 times, then  $x$  is connected to 3 vertices out of four consecutive ones in  $C_t$ . Out of these three vertices two have to be consecutive, contradicting the maximality of  $C_t$ . If  $\nexists x \in \Gamma_1$  which is covered 3 times, then there must be (at least) three vertices, say,  $x_1, x_2, x_3 \in \Gamma_1$  which are covered twice by  $\bigcup_{j=1}^4 N_{NEW}(v_{4i+j}, \Gamma_1)$ . Consider the bipartite graph  $G_i$  with parts  $A_i = \{v_{4i+j} : j = 1, \dots, 4\}$  and  $B = \{x_1, x_2, x_3\}$  with the edges defined by  $T_i$ . All vertices in  $B$  are of degree 2. A trivial case analysis shows that there always exists a rainbow matching formed by two edges of  $G_i$ .  $\square$

So we may assume that for each  $i$ , inequality (8) does not hold. But then for the number of new edges  $|E_{new}[\Gamma_1, C_t]|$  between  $\Gamma_1$  and  $C_t$  by Claim 2

$$|E_{new}[\Gamma_1, C_t]| \leq \frac{t}{4}(\deg_{NEW}(x'_1, R') + 2) \quad (9)$$

holds. Combining estimates (7, 9) we get

$$\deg_{NEW}(x'_1, R') \cdot (n - \frac{3t}{2} - 3d - 6) \leq \frac{t}{4}(\deg_{NEW}(x'_1, R') + 2),$$

which implies (dividing by  $\deg_{NEW}(x'_1, R')$  and using (3))

$$t \geq \frac{4n}{7} - \frac{12d + 24}{7} - \frac{2t}{7(n - \frac{3t}{2} - 2d - 2)}. \quad (10)$$

Here for the last term we have

$$\frac{2t}{7(n - \frac{3t}{2} - 2d - 2)} \leq \frac{8}{7}. \quad (11)$$

Indeed, if (11) does not hold, then we have

$$t > \frac{4n}{7} - \frac{8d + 8}{7}, \quad (12)$$

i.e. again we have a lower bound similar to (5). Thus otherwise from (10) we get

$$t \geq \frac{4n}{7} - \frac{12d + 32}{7}. \quad (13)$$

**Case 2:** Assume that no pair of vertices  $y_1, y_2$  exists within distance  $d$  along the cycle that are adjacent in two different new colors to two different vertices, say  $x_1, x_2 \in R$ . This implies easily that in each interval of length  $d$  along the cycle there is at most one vertex  $x$  with  $\deg_{NEW}(x, R) \geq 3$ . Therefore, the number of edges in new colors between  $C_t$  and  $R$  is at most

$$\frac{t}{d}|R| + 2t \leq \frac{2t}{d}|R|,$$

since  $2d \leq n/4 \leq |R|$  (using (1)).

Thus, if we denote by  $B$  the set of those bad vertices  $x \in R$  for which

$$\deg_{NEW}(x, C_t) \geq \frac{2t}{\sqrt{d}},$$

then we have

$$|B| \frac{2t}{\sqrt{d}} \leq \frac{2t}{d}|R| \quad \text{i.e.,} \quad |B| \leq \frac{|R|}{\sqrt{d}}.$$

Set  $R^* = R \setminus B$ . We have  $|R^*| \geq (1 - \frac{1}{\sqrt{d}})|R|$ . Moreover,  $R^*$  is almost complete in new colors. For every  $x \in R^*$  we have:

$$\begin{aligned} \deg_{NEW}(x, R^*) &\geq |R| - 1 - \frac{2t}{\sqrt{d}} - \frac{|R|}{\sqrt{d}} \geq |R| - \frac{3t}{\sqrt{d}} - \frac{|R|}{\sqrt{d}} \geq \\ &\geq |R| \left(1 - \frac{10}{\sqrt{d}}\right) \geq |R^*| \left(1 - \frac{10}{\sqrt{d}}\right), \end{aligned} \tag{14}$$

where the third inequality is equivalent (through  $|R| + t = n$ ) to  $t \leq 3n/4$ . We can assume this, otherwise we have nothing to prove.

**Lemma 1.** *Suppose  $k, l$  are given integers with  $l < k/2$  and  $G$  is a properly edge colored  $k$ -vertex graph with minimum degree at least  $k/2 + l$ . Then an arbitrary pair of vertices  $x_1, x_2 \in V(G)$  can be joined by a rainbow path of length at least  $\frac{2l}{3}$ .*

**Proof.** Starting at  $x_1$ , build a greedy path by extending the current endpoint  $y \neq x_1$  with an edge  $yz$  such that  $z \neq x_2$  and  $yz$  has a color not used on the current path. Assume that at a certain point we have  $P = \{x_1, \dots, y\}$ . Call a color new, if it does not appear on  $P$ . Set  $Q = V(G) \setminus (V(P) \cup \{x_2\})$  and  $m = k/2 + l - 2|P|$ . Observe that  $\deg_{NEW}(y, Q) \geq m$  and  $\deg_{NEW}(x_2, Q) \geq m$ . Thus, if

$$2m = k + 2l - 4|P| > |Q| = k - |P| - 1,$$

i.e., if equivalently  $\frac{2l+1}{3} > |P|$ , then  $M = N_{\text{new}}(y, Q) \cap N_{\text{new}}(x_2, Q) \neq \emptyset$ . Thus with  $w \in M$ , the path  $P^+ = Pwx_2$  is a rainbow path from  $x_1$  to  $x_2$  so there exists a path  $P^*$  such that

$$|P^*| = \left\lfloor \frac{2l+1}{3} \right\rfloor - 1 + 2 \geq \frac{2l}{3},$$

as desired.  $\square$

Choose  $G$  as the subgraph induced by the edges with new colors in  $R^*$ , set  $k = |R^*|$  and notice that using Lemma 1 and (14) we can join an arbitrary pair of vertices in  $R^*$  by a rainbow path in all new colors of length at least

$$|R^*| \left( \frac{1}{3} - \frac{20}{3\sqrt{d}} \right) \geq |R| \left( 1 - \frac{1}{\sqrt{d}} \right) \left( \frac{1}{3} - \frac{20}{3\sqrt{d}} \right) \geq |R| \left( \frac{1}{3} - \frac{7}{\sqrt{d}} \right).$$

For some  $\ell$ , move the colors of the edges of the path  $v_1, \dots, v_{l+1}$  along the cycle from  $OLD$  to  $NEW$ , now  $|NEW| \geq n - t - 1 + \ell$ . If

$$n - t - 1 + \ell \geq t + |B| + 3 \geq t + \frac{|R|}{\sqrt{d}} + 3 \quad \text{i.e.,}$$

$$\ell \geq 2t - n + \frac{|R|}{\sqrt{d}} + 4, \tag{15}$$

then  $v_1$  and  $v_{l+1}$  both send at least 3 new colors to  $R^*$  out of which we can find a rainbow matching of two edges, say,  $(v_1, x_1)$ ,  $(v_{l+1}, x_2)$ , where  $x_1, x_2 \in R^*$ . But if in addition

$$\ell \leq |R| \left( \frac{1}{3} - \frac{7}{\sqrt{d}} \right), \tag{16}$$

then we could substitute the path  $\{v_1, \dots, v_{l+1}\}$  by the path  $\{v_1, x_1, P, x_2, v_{l+1}\}$  of length  $\ell + 2$ , where  $P$  is a path of length  $\ell$  joining  $x_1$  and  $x_2$  which must exist by Claim 1, a contradiction. Therefore no  $\ell$  satisfies both (15) and (16), so we may assume

$$|R| \left( \frac{1}{3} - \frac{7}{\sqrt{d}} \right) < 2t - n + \frac{|R|}{\sqrt{d}} + 4,$$

and substituting  $|R|$  by  $(n - t)$  we conclude that

$$\begin{aligned} 7t &> 4n + \frac{24t}{\sqrt{d}} - \frac{24n}{\sqrt{d}} - 12 > 4n - \frac{24n}{\sqrt{d}} - 12, \quad \text{i.e.,} \\ t &> \frac{4n}{7} - \frac{24n}{7\sqrt{d}} - \frac{12}{7}. \end{aligned} \tag{17}$$

We finish the proof by observing that with our choice of  $d$  and  $n_0$  (see (1)) all the obtained lower bounds on  $t$  (namely (5), (12), (13) and (17)) are at least  $(4/7 - \varepsilon)n$ .  $\square$



## References

- [1] S. Akbari, O. Etesami, H. Manini, M. Mahmoody, On rainbow cycles in edge colored complete graphs, *Australasian Journal of Combinatorics*, 37 (2007), 33-42.
- [2] M. Albert, A. Frieze, B. Reed, Multicoloured Hamilton cycles, *Electronic Journal of Combinatorics*, 2 (1995), #R10.
- [3] L.D. Andersen, Hamilton circuits with many colours in properly edge-colored complete graphs, *Math. Scand.*, 64 (1989), 5-14.
- [4] A.E. Brouwer, A.J. de Vries, R.M.A. Wieringa, A lower bound for the length of partial transversals in a Latin square, *Nieuw Arch. Wisk.*, (3) 24 (1978), 330-332.
- [5] P. Erdős, J. Nešetřil, V. Rödl, Some problems related to partitions of edges of a graph, in *Graphs and Other Combinatorial Topics*, Teubner, Leipzig (1983), 54-63.
- [6] A. Frieze, M. Krivelevich, On rainbow trees and cycles, *Electronic Journal of Combinatorics*, 15(1) (2008), #R59.
- [7] A. Frieze, B. Reed, Polychromatic Hamilton cycles, *Discrete Mathematics*, 118 (1993), 69-74.
- [8] A. Gyárfás, M. Mhalla, Rainbow and orthogonal paths in factorizations of  $K_n$ , *Journal of Combinatorial Designs*, 18(3) (2010), 167-176.
- [9] G. Hahn, C. Thomassen, Path and cycle sub-Ramsey numbers and an edge-colouring conjecture, *Discrete Mathematics*, 62 (1986), 29-33.
- [10] P. Hatami, P.W. Shor, A lower bound for the length of a partial transversal in a Latin square, *Journal of Combinatorial Theory, Series A*, 115 (2008), 1103-1113.
- [11] M. Maamoun, H. Meyniel, On a problem of G. Hahn about coloured Hamiltonian paths in  $K_{2n}$ , *Discrete Mathematics*, 51 (1984), 213-214.
- [12] H.J. Ryser, Neuere Probleme der Kombinatorik, in: Vorträge über Kombinatorik, pp. 69 - 91, a collection of papers presented at the Mathematischen Forschungsinstitute, Oberwolfach, July 24 - 29, 1967.
- [13] P.W. Shor, A lower bound for the length of a partial transversal in a Latin square, *Journal of Combinatorial Theory, Series A*, 33 (1982), 1-8.

- [14] D.E. Woolbright, An  $n \times n$  Latin square has a transversal with at least  $n - \sqrt{n}$  distinct symbols, *Journal of Combinatorial Theory, Series A*, 24 (1978), 235-237.